Advanced Mathematical Methods for Telemetry Analysis

Dr. Christopher Landauer
The Aerospace Corporation

Computer Systems Division
13 January 2015

2015 Spacecraft Flight Software Workshop

27 Oct 2015, JHU / APL

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- **Does not contain US Export Controlled information**

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Summary

- Spacecraft telemetry delivers an enormous amount of data.
  - *Usually and mostly about the hardware.*
- Advanced mathematical methods have the potential
  - *To detect weak early indications or precursors of anomalies,*
  - *To identify unexpected correlations in system measurements,*
  - *To identify major hardware degradation in advance.*
- These capabilities may lead to earlier and better behavior predictions
  - *For ground-based analysis of spacecraft operation.*
  - *Eventually, maybe better on-board anomaly prediction / detection,*
    - Even correction of some anomalies
- Experimentation is needed to understand what is and is not detectable.
  - *How far in advance anomalies can be detected.*
  - *Which advanced mathematical methods are more effective.*
  - *What other data would be useful.*

The big problem is software.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Spacecraft Telemetry Issues

- Classic analyses
  - *Acceptable intervals and short-term trends*
- Rarely
  - *Cross-mission norms and anomalies*
- Predictive models based on planned mission profile, and history of similar programs, satellites, or components
- In earlier papers,
  - *We advocated extensive software telemetry.*
  - *We also advocate more telemetry for detailed instrumentation.*
    - MUCH more!
    - We explain later how this choice need not be the bandwidth problem it might seem to be

Extend telemetry analyses to more software, cross-missions and longer times

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Software Caused Failures

- NASA's Mars Climate Orbiter (23 September 1999):
  - *Different developers used different physical units in thruster commands.*
- NASA's Mars Polar Lander (03 December 1999):
  - *Software did not distinguish leg deployment from landing shocks.*
- ESA's Huygens Probe (14 January 2005):
  - *One communication channel (of two) was never commanded to turn on.*
- All three have something in common:
  - *They are attributable to insufficiently careful software or software design.*
    - In the ESA case, it was a CONOPS problem that more carefully designed software could have detected.
  - *Something quite simple went wrong quite disastrously.*
- The problem is that there are many millions of simple things that can go wrong.
  - *Similar issues occur in some military satellite failures.*
  - *Descriptions of these ones are more widely accessible.*

We have no development methods capable of managing this complexity.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Software Telemetry

- Each of these issues could have been found with more extensive software telemetry.

- Different context of reuse should have led to scrutiny of value ranges.
  - *"It has flown successfully before'' is a wholly inadequate justification.*
  - *Values can be reported and compared to expected value ranges.*
- Simple on-board sanity checks should occur for all critical parameters.
  - *This is where the numbers hit the processors.*
  - *This is where they should be checked.*
- Ditto
  - *Every value put into a database should be checked.*
  - *Telemetry should announce all values received prior to execution.*

- Auditable chain of custody for every numerical parameter.
  - *Not all problems are software.*

These steps will help; they are far too seldom implemented, and by themselves, are not enough.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Advanced Mathematical Methods

- To provide perspective across:
  - *Long time periods*
  - *Measurements*
  - *Missions*
- Some example kinds of complex detailed analyses
  - *Robust Statistics*
  - *Grammatical Inference*
  - *Event Pattern Correlation*
  - *Time Series Analysis*
    - Hidden Markov Models
    - Dynamic Time Warping
  - *Fractal Dimension*
  - *Dimension Reduction*
  - *Manifold Discovery*
  - *Topological Data Analysis*

Many methods exist; we chose a few of them.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Some Example Applications

- Temporal characteristics detection
  - *``Regime change''*
    - Mission phases are planned well in advance
    - Expectable behavior can be predicted
      - *Statistics can be predicted,*
      - *Different for each phase*
  - *Signal texture definition and identification*
    - Expected noise level and pattern, general trend
    - Fractal dimension for change detection
  - *Each hardware measurement has relatively well-known known degradation and failure characteristics.*
    - Software is more difficult, because we know much less about degradation and failure of software components.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Some Example Applications (cont.)

- Correlations, expected and otherwise
  - *Time delayed correlations can detect potential causalities.*
  - *Distinguishing causality from correlation depends on deviation correlations.*
- Behavior in abstract trajectory spaces
  - *Long term behavior of measurements over time and correlations thereof*
  - *Dimension reduction to identify nominal behavior and first excursions.*
    - Computing expectable constraints

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Some Practicalities

- Is this too much telemetry?
  - *We think the increased information outweighs the processing burden and decreases risk.*
  - *The increased information does not necessarily increase required bandwidth.*

- Compressed sensing for telemetry suites
  - *More telemetry, less bandwidth*
  - *Need to trust the compression process*
    - Process to detect measurement sequences outside the expected space
    - Also an occasional non-compressed item,
      - *used like tracers to verify compression process*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Some Practicalities (cont.)

- This is an example of dimension reduction
  - *Define the expected space of measurement sequences*
    - For each measurand separately (and some together)
  - *Use manifold discovery to determine what part of the space actually gets used*
    - This is also a prediction derived from nominal behavior
      - *And expectable anomalies*
    - Construct a low-dimensional manifold in the measurement sequence space
    - This is a common model with fewer parameters.
  - *Map from actual measurement sequences to the smaller manifold*
    - Fewer dimensions means fewer bits to send
    - Original measurement reconstructed from the values received
      - *and the known common model*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Conclusions and Prospects

- Exciting prospects for extracting more comprehensive information from telemetry
  - *Using a host of advanced mathematical methods (see the Appendix)*
- These computations need some serious experimentation for feasibility and location
  - *How much can be done on-board? on-line?*
  - *How much can only be done off-line or retrospectively?*
  - *How much isn't particularly helpful?*
  - *How much might be helpful with further development or research?*
- We need to collect much more extensive software telemetry for tracking the internal progress of computations.
  - *Happily, we can do that on the ground.*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Conclusions and Prospects (cont.)

- We need to experiment with software telemetry also
  - *What parts of the code are the most important to measure?*
    - decision points and other branches
    - function calls and parameters
    - interrupts and pre-emption
    - fixed point arithmetic computations
    - ...what else?

- We need new and much more extensive stress testing of units and components (to limit damage propagation or escalation).
  - *Can we prevent small errors from leading to catastrophic failure?*
  - *Can we prevent most errors from leading to catastrophic failure?*
  - *Study of error propagation through complex numerical calculations*
  - *Study of error propagation through non-numerical calculations*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Questions?

Appendix: Definitions and Examples of Selected Methods

- List of Selected Methods
  - *Robust Statistics*
  - *Grammatical Inference, Event Pattern Correlation*
  - *Time Series Analysis, Hidden Markov Models, Dynamic Time Warping*
  - *Fractal Dimension, Large Dimensions*
  - *Dimension Reduction, Random Projections*
  - *Manifold Discovery, Local Linear Embedding, ISOMAP*
  - *Topological Data Analysis*
- Define for each area
  - *What it is about*
    - Not formal definitions; those are available
  - *What it can do*
    - What it has already been used for, at least in a lab
  - *What it might be able to do*
    - With some extension and further development

This is a short informal introduction. Details can be found elsewhere.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Robust Statistics

- What it is
  - *Statistical computations that are not dependent on distribution assumptions*
  - *(every empirical distribution is usually assumed to be Gaussian; many are)*

- What it can do
  - *Identify outliers, identify vulnerabilities of any statistical summary*
  - *(what fraction of bad data can cause bad results)*

- What it might be able to do
  - *More reliable identification of noise (structured and not)*
  - *Better (less noisy) summaries of data*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Robust Statistics Example

- The example is about a sample set (where we consider different values for x)

  {0, 3, 7, 8, x}.

- We can compute that the mean of this set is

  $m = 3.6 + 0.2 * x,$

- and that the variance is

  $v = 11.44 - 1.44 * x + 0.16 * x^2.$

- As x gets large, both of these increase without bound.

- The **breakdown point** of an estimator is the proportion (or number) of incorrect observations an estimator can handle before giving an (arbitrarily) incorrect result.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Robust Statistics Example (cont.)

- The robust ``figure skating'' rule is to omit the largest and smallest values.

- Then whenever x is larger than 8, the mean is 6 and the variance about 4.667
  - *The breakdown point of this method is 1 if the statistic is one-sided*
  - *(a second very large value will be retained),*
  - *and the method can sometimes works with 2 outliers*
  - *(when one is excessively large and one excessively small).*

- The **influence functions** for m and v are the algebraic expressions above, showing how m and v depend on x.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

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Grammatical Inference

- What it is
  - *Infer structure rules (syntax) from item sequences*
  - *Rules have the form LHS ==> RHS*
    - RHS is a sequence of terminal and non-terminal symbols
      - *terminal symbols correspond to items, so they do not change*
      - *non-terminal symbols correspond to classes of structured items*
    - LHS is also, with various restrictions that determine a classification
      - *PSG = Phrase Structure Grammar (at least one non-terminal)*
      - *CSG = Context Sensitive Grammar (one non-terminal in LHS changes to a sequence in RHS)*
      - *CFG = Context-Free Grammar (only one non-terminal)*
      - *RG = Regular Grammar (further restrictions on grammar)*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Grammatical Inference (cont.)

- What it can do
  - *Find compact rules for complex hierarchical behavior structures*
    - Structure recognition (parsing)
      - *Make sure the language contains enough correct sets*
    - Generation of random sets
      - *Make sure the language contains few incorrect sets*

- What it might be able to do
  - *New mathematics to define partially ordered set grammars*
  - *(see Event Patterns below)*
    - Inference of pattern grammars from complex data

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Grammatical Inference Example

- Start with a large finite set of strings (sequences of characters)
- Problem is inherently ambiguous
  - *Many grammars generate languages that contain exactly the same strings*
  - *Usually want the smallest one*
- Sample inference rules (each makes a new non-terminal symbol and substitutes it into the strings)
  - *If A . B occurs a lot, infer new rule Z ==> A . B*
    - (and replace A . B with Z wherever it occurs)
  - *If X . A . Y and X . B . Y occur a lot, infer new rule Z ==> A | B*
    - (and replace A and B with Z wherever they occur)
    - (we can retain the context X _ Y in the rule if it is a CSG)
  - *If sequences of A's occur a lot, infer new rules Z ==> A and Z ==> Z . A*
    - (and replace sequences of A's with Z wherever they occur)
    - (or just Z ==> A+ if RG)
- Problem: cannot infer CFG with only positive examples
- Inferring recursion is difficult, but useful (it distinguishes CFG from RG)

Event Pattern Correlation

- What it is
  - *Matching complex occurrence patterns separated in space and time*
  - *General partially ordered sets, instead of merely sequences*

- What it can do
  - *Make connections from similar structure*

- What it might be able to do
  - *Highly scalable algorithms for correlations among many sequences*
    - Want something with complexity less than quadratic if possible
  - *New mathematics of non-sequence correlation*
    - Extension of previous algorithms for grammatical inference and parsing
  - *Much more flexible event pattern detection and correlation*
    - Then add a temporal component

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Event Pattern Correlation Example

- Identifying TCP connection attempts
  - *Client side: Start in Initial state*
    - Send SYN
      - *SYN_SENT: Wait for SYN and ACK (or close from client)*
    - Receive SYN and ACK
    - Send ACK
      - Connection Established
  - *Server side: Go to LISTEN mode (Wait for SYN)*
    - Receive SYN
    - Send SYN and ACK
      - *SYN_RCVD: Wait for ACK*
    - Receive ACK
      - Connection Established
    - Receive RST, go to Initial
- These message sends can be detected
- Many coordination protocols can be identified this same way

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Time Series Analysis

- This topic is too big, consider Hidden Markov Models (HMM), Dynamic Time Warping (DTW)
- What HMM is
  - *Data implies maximum likelihood model parameters*
    - Model implies sample data
  - *EM algorithm is the iteration of this process*
- What it can do
  - *Determine certain kinds of hidden structure (HMMs are an example)*
  - *Impute missing data*
  - *Compute sensitivity of models to imputed data*
- What it might be able to do
  - *Algorithms to speed up EM iteration*
  - *Other hidden structures besides periodicity (via Fourier analysis)*
    - Cascaded structures (e.g., periodicity depends on mode, mode changes according to Finite State Machine or Markov Model

Hidden Markov Model Example

- Long sequence of English text (words only)
- Assume two-state Markov chain and size 27 (letters + space) output alphabet
- Compute Maximum Likelihood estimate for transition probabilities and output maps
- Apply forward-backward algorithm (linear in the total length of the string)
  - *Compute transition matrix and output mapping that maximize the likelihood of the observed string*
  - *Then the states clearly correspond to vowels and consonants*
    - Letters y, w, and r are slightly more ambiguous than others
- Similar results hold for up to 12 states
  - *Phonetically meaningful categories derived from ordinary English spelling*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Dynamic Time Warping

- What DTW is
  - *Matching one time series to another, with different and possibly variable time progressions*

- What it can do
  - *Match speech patterns to templates*
  - *Match business rhythms of different duration*

- What it might be able to do
  - *Extend to a plane to match images in a new registration style*
    - Local distortion
  - *Match computers running the same code by output signals or by heat generation / power use*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Fractal Dimension

- What it is
  - *Determine fractal dimension of a set, which determines minimum guaranteed embedding dimension (using tuple map)*

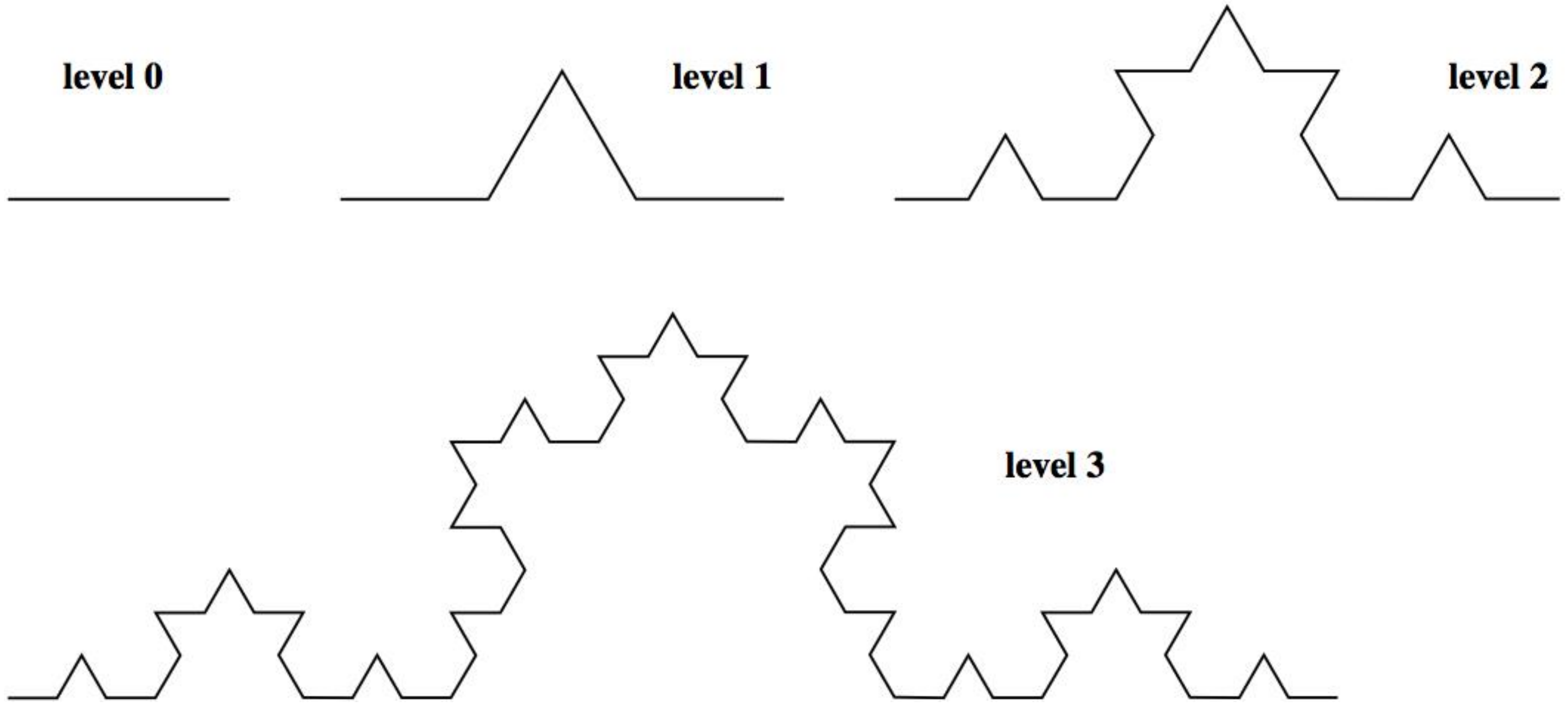
- What it can do
  - *Bound the computation of complex dynamics*
    - If d = fractal dimension,
    - then any m > 2 * d + 1 is sufficient for embedding the dynamics
    - (and smaller dimensions can work)

- What it might be able to do
  - *More reliable computation of fractal dimension*
  - *Much faster computation of a close upper bound*
  - *Hard to spoof*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Fractal Dimension Example

- Koch curve, dimension log 4 / log 3 ~ 1.26186
- The dimension of any *self-similar* figure can be computed fairly readily.



chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Things are Tricky in Many Dimensions

- The n-volume V_n of the unit n-sphere decreases rapidly to zero as n increases
    - V_0 = 1, V_1 = 2, V_2 = pi, V_3 = (4/3) * pi (initial values, known values)
    - V_{n} / V_{n-2} = 2*pi / n for n >= 2,
  - *which ratio monotonically decreases for all n >= 1,*
  - *and the n-volume V_n decreases for n >= 7 (after increasing for n < 7)*

- If we inscribe an n-sphere in an n-cube, the n-volume ratio goes rapidly to zero
  - *Eventually, other unit n-spheres can be packed into the corners disjointly*
  - *(Martin Gardner devoted an entire column to these issues)*

- The volume of an n-annulus (n-sphere shell), with inner radius = 90% outer radius is almost the same as the volume of the n-sphere, and the ratio goes to 1 as n increases
    - ratio = 1 - (0.9) ^ n

Almost everything happens at the boundaries or in the corners.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Dimension Reduction

- What it is
  - *Structure preserving map from high-dimensional space into low-dimensions*
    - Different structures need different algorithms to compute different maps
      - *Nearby pair distances*
      - *Local linear relations*
      - *Geodetic distances*
      - *Tangent space alignments*
    - Generally choose how many dimensions in advance
      - *and see later how well it worked*
    - Could conceivably us an intrinsic dimensionality like Fractal Dimension

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Dimension Reduction (cont.)

- What it can do
  - *Reduces computational effort, sometimes a lot (millions or more dimensions to hundreds or even just dozens)*
  - *Images, video and other signals, abstract trajectories and other event patterns are all high-dimensional data*
  - *Relationship and other connection patterns are high-dimensional data*
  - *Trajectories of other values can be considered as elements in a high-dimensional abstract trajectory space*

- What it might be able to do
  - *All of these can be simplified with some structure preserved*
    - Usefully? We don't know yet
  - *More different kinds of preserved structures*
  - *Data-dependent algorithm choice*
  - *Faster and more robust algorithms (better source data uncertainty management)*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Dimension Reduction Example

- Non-linear map (J.Sammon)
- Given a point set X in Real ^ D for large D, we want a mapping
    - f : X --> Real ^ d
  - *usually with d=2 or d=3 and minimum **distortion***
- Compare d(f(x),f(y)) to d(x,y) over all pairs x, y in X
  - *Distortion is sum of squared distance differences*
    - (d(f(x),f(y)) - d(x,y)) ^ 2
  - *Normalized by original distance d(x,y) to emphasize preserving small distances*
  - *Each point in X is mapped separately, so there are d * |X| variables*
- Simple (if time-consuming) optimization problem
  - *Many global minima (every permutation of X, every isometry of Real ^ d), so we need some criterion to limit translations*
    - Maybe one point is origin, always mapped to 0
    - Maybe sum of images is 0
    - Maybe choose d+1 points to anchor to a particular isometry in Real ^ d

Some Theorems

- Johnson-Lindenstrauss Theorem

  - *Any n point set in any Euclidean space can be embedded in k dimensions*
    - $k = O(\log n / \varepsilon ^ 2)$
  - *without distorting any pair distance by more than a factor of*
    - $1 \pm \varepsilon$
  - *for any ε with $0 < \varepsilon < 1$.*

  - *The issue is to find the smallest useful value of the constant for k.*

  - *The weird part is that Random Linear Projections almost always work.*
    - There is lots of research on how well restricted classes of projections work.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

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Some Theorems (cont.)

- Concentration of Measure Theorem

  - *Write S^{n-1} for the (n-1)-sphere in n-space, { x in Real ^ n | |x| = 1}.*

  - *If f : S^{n-1} --> Real is 1-Lipschitz*
    - (i.e., |f(x) - f(y)| <= |x - y| for all x, y),
  - *and if s() is a normalized measure on the sphere*
    - (i.e., s(A) is the ratio of the area of A to the area of the sphere),
  - *Then*
    1. There is at least one real number M (called a median for f) for which
       - *s({x | f(x) <= M}) = 1/2 = s({x | f(x) >= M}),*
    2. s({x | |f(x) - M| > ε}) <= 2 * exp(- n * ε ^ 2 / 2)
       - *(i.e., f is very seldom far from M, so it is almost constant)*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Manifold Discovery

- What it is
  - *Identify low-dimensional manifold in high-dimensional data (example of dimension reduction)*
- What it can do
  - *Discover unexpected and previously unknown constraints*
    - Local linear structure
    - Other neighborhood properties
    - Connected distance properties
- What it might be able to do
  - *Better management of data content and noise reduction*
    - Compressed sensing
  - *Dynamic characteristics of high-dimensional images and signals*
    - Track changes, notice trends, identify fluctuations
  - *Allows interpolation for new data points*
  - *Hard to spoof*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

LLE, ISOMAP

- Two methods for Dimension Reduction
- LLE = Local Linear Embedding
  - *Local structure is defined by the best linear approximation of a point by its near neighbors, recorded in a coefficient matrix*
  - *Then the projection is the set of points that best preserves the coefficient matrix*
- ISOMAP
  - *Local structure is defined as the graph of its near neighbors;*
    - Distance is measured in edge lengths
- There are many others; all depend on a choice of neighborhood
  - *Distance or count threshold for size of neighborhood*
  - *Few or no studies of the effects of such choices*
- Concentration of measure theorem makes all distance-based methods suspect
  - *Examples can be found in the literature*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Topological Data Analysis

- What it is
  - *Fast homology computations for complex high-dimensional structures*
  - *(mapping complex geometric and topological structures to computable algebraic structures)*
- What it can do
  - *Summarize complex connectivity structures at all scales, robust to noisy data*
- What it might be able to do
  - *Compute sensitivity of structure to noise level*
  - *More effective image pyramid (less compression in slowly varying parts of the image)*
  - *Detect fundamental change in organizational relationships and other structures*
  - *Detect subtle differences among correct, erroneous, and fake data*
  - *The ``tidy set'' is a significant computational reduction (size and therefore time).*

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Second List

- The lead candidates for a next round of study are:
  - *Uncertainty propagation*
  - *Identification of missing data and its potential utility to guide collection requests*
  - *Object extraction from images and signals*
  - *Flow mapping and non-explicit differential equations*
  - *Natural Language Processing (in multiple languages)*
  - *Graph theoretic analyses*
  - *Spatial-Temporal Reasoning*
  - *Advanced steganography*

There are, of course, many potentially useful methods we have not listed here.

chris.landauer@aero.org
Software Systems Analysis Dept./Software Engineering Subdivision

Further Information

- For questions and / or comments, please contact the author

chris.landauer@aero.org